Problem 8: Cow Pinball
11 Points

Problem ID: `pachinko`
Rank: 3

Introduction

Big Ben and Bessie the Gaur have returned to the CALICasino in Las Vegas, eager to live out their big screen dreams. After having listened to the Who's famous song from their 1969 album *Tommy*, Pinball Wizard, they dash to the pinball machines, eager to fork over their mortgage and children's college funds for a chance to win big.

As they flip away their dreams on the pinball poor-inator over and over, the casino employs you to figure out how much money they can probabilistically prune off the pessimistic prospects of our protagonists.

Problem Statement

The paths a ball can take in a pinball machine is represented by a rooted tree with $\mathbf{N}$ vertices labeled 1 to $\mathbf{N}$, where the root is labeled 1 and edges point away from the root. The parents of each vertex in the tree are given by the sequence $\mathbf{P_2}, \ldots, \mathbf{P_N}$, where $\mathbf{P_i}$ is the parent of the vertex labeled i. There is also a flipper, which adds an additional edge from vertex $\mathbf{S}$ to vertex $\mathbf{E}$, possibly creating cycles.

In a game of pinball, the ball starts at the root. At each vertex, the ball randomly chooses an outgoing edge to follow with **uniform probability**. What is the expected number of edges the ball will follow before it arrives at a vertex with no outgoing edges?

Input Format

The first line of the input contains a single integer $\mathbf{T}$ denoting the number of test cases that follow. For each test case:

- The first line contains three space-separated integers $\mathbf{N}$ $\mathbf{S}$ $\mathbf{E}$ where:
  - $\mathbf{N}$ denotes the number of vertices
  - $\mathbf{S}$ and $\mathbf{E}$ denotes the extra edge from the vertex labeled $\mathbf{S}$ to the vertex labeled $\mathbf{E}$.
- The next line contains $\mathbf{N}$ - 1 space-separated integers $\mathbf{P_2}$, …, $\mathbf{P_N}$, representing the pinball machine.
  - $\mathbf{P_i}$ denotes the parent vertex of the i-th vertex.

Output Format

For each test case, output the expected number of edges that will be traversed in a game with **an absolute error of within 10^{-5}**.

Constraints

$1 \leq \mathbf{T} \leq 10$
$2 \leq \mathbf{N} \leq 1000$
$1 \leq \mathbf{S}, \mathbf{E} \leq \mathbf{N}$
It is guaranteed that the solution is finite.

Sample Test Cases

Sample Input

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```
5
5 4 3
1 2 1 4
9 6 3
1 1 2 2 5 1 7 6
3 2 2
1 2
3 2 1
1 2
23 7 1
1 2 3 4 5 6 7 4 9 9 9 10 10 12 12 12 16 16 14 14 21 21
```

Sample Output

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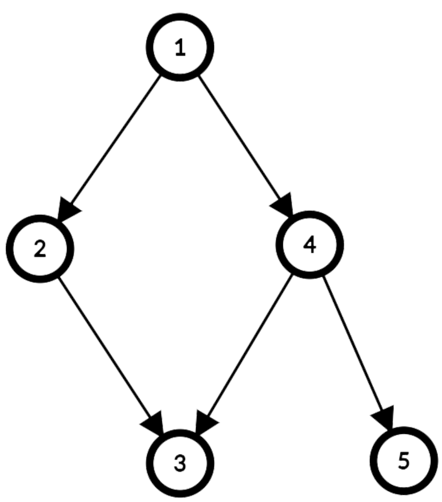
```
2.0
2.0
3.0
4.0
8.6851851
```

Sample Explanations

Test Case #1:

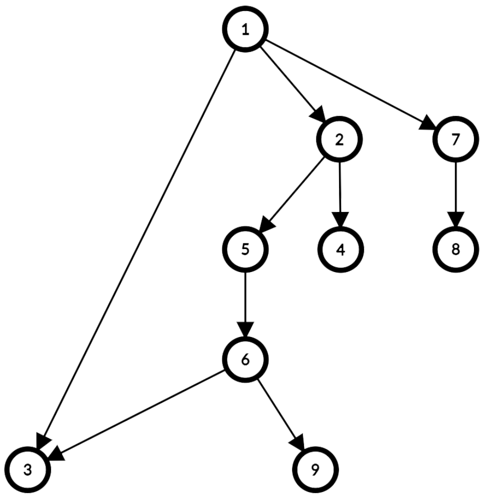
The pinball machine looks like this:



The tree is formed by 5 vertices, with the flipper edge connecting vertex 4 to vertex 3. Since all paths that start at vertex 1 and end in a leaf are of length 2, the expected amount of edges traversed is 2.0.

Test Case #2:

The pinball machine looks like this:



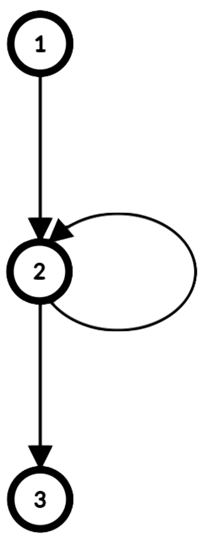
The tree is formed by 9 vertices and the flipper edge connects vertex 6 to vertex 3. All possible paths starting from vertex 1 are:

- 1→3 with 33.33% chance
- 1→2→4 with 16.67% chance
- 1→7→8 with 33.33% chance
- 1→2→5→6→3 with 8.33% chance
- 1→2→5→6→9 with 8.33% chance

Overall, the expected amount of edges traversed is 2.0.

Test Case #3:

The pinball machine looks like this:



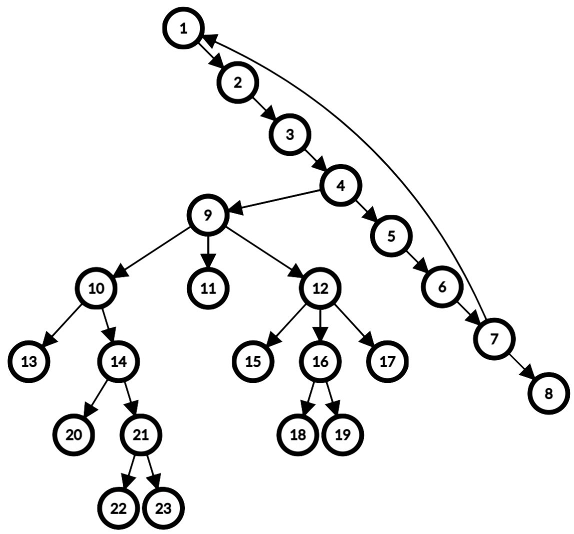
In the third case the tree is just a line consisting of 3 vertices and the flipper goes from vertex 2 to itself, creating a cycle in the graph. Some paths starting from vertex 1 are:

- 1→2→3 with 50% chance
- 1→2→2→3 with 25% chance
- 1→2→2→2→3 with 12.5% chance

In total, the expected amount of edges traversed is 3.0.

Test Case #5:

The pinball machine looks like this:



A possible run of the pinball game would be the following:
1→2→3→4→5→6→7→1→2→3→4→5→6→7→1→2→3→4→9→12→16→19, which has a 0.1736% chance of occuring and traverses 21 edges.

The expected edges traversed in this graph are approximately 8.847222.

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