## Problem 9: maushold 11 Points

Problem ID: maushold
Rank: 3

## Introduction



When your Pokémon's Hit Points drop below zero, your Pokémon faints! So it is very important to know if your Pokémon will survive the next hit or not. That is why most players use the Pokémon Showdown! Damage Calculator, as it gives you the exact damage that the opposing Pokémon will deal. Note that the damage formula involves an internal dice roll, so depending on your luck a Pokémon's move will deal less or more damage. This calculator then gives you the probability of your Pokémon fainting after receiving one move.

Even though damage calculators that use Pokémon Showdown!'s algorithm are really helpful, they are actually inaccurate! Some moves involve attacking multiple times, and the damage calculator combines all hits into one, giving a rough estimate. For example, if a move involves hitting twice, the calculator will treat the move as if it was just one hit but twice as strong.

However, imprecisions occur with moves such as Maushold's Population Bomb, which can hit up to ten times! Just imagine the imprecision that would incur if Maushold hit up to $10^{12}$ times!

## Problem Statement

Your Pokémon has H Hit Points. Maushold's Population Bomb attacks hits $\mathbf{N}$ times, and the damage each hit deals is randomly chosen from a list of $\mathbf{M}$ numbers $\mathbf{R}_{\mathbf{1}} \mathbf{R}_{\mathbf{2}} \ldots \mathbf{R}_{\mathrm{M}}$. Calculate the exact probability that your Pokémon will faint after receiving Maushold's attacks (i.e. all hits combined deal at least $\mathbf{H}$ points of damage). Each of the $\mathbf{R}_{\mathbf{1}} \mathbf{R}_{\mathbf{2}} \ldots \mathbf{R}_{\mathrm{M}}$ damage rolls are equally likely to occur, and each hit is independent of the others. The probability of your Pokémon fainting after less than $\mathbf{N}$ hits should additionally be included in your answer.

Since the probability of your Pokémon fainting can always be represented by an irreducible fraction $p / q$, compute $p \times q^{-1} \bmod 998244353$. In the case that the probability is 0 , just print 0 . We define " $q^{-1}$ mod 998244353 " to be the modular inverse of $q$, a number such that $q \times q^{-1}$ equals 1 mod 998244353 .

## Input Format

There is only one test case for each test case file:

- The first line contains three space-separated integers H N M denoting the number of Hit Points, the number of times that Maushold hits and the number of different damage rolls, respectively.
- The next line contains $\mathbf{M}$ space-separated integers $\mathbf{R}_{\mathbf{1}} \mathbf{R}_{\mathbf{2}} \ldots \mathbf{R}_{\mathbf{M}}$, denoting the number of Hit Points that each damage roll takes away from your Pokémon.


## Output Format

For each test case, output a single integer $p \times q^{-1}$ modulo 998244353, where $p / q$ is the probability of your Pokémon fainting after receiving $\mathbf{N}$ hits. It can be proved that $q$ is never congruent to 0 modulo 998244353.

## Constraints

Time limit: 4 seconds.
$1 \leq \mathbf{H} \leq 2000$
$1 \leq \mathbf{N} \leq 10^{12}$
$1 \leq \mathbf{M} \leq 200000$
$0 \leq \mathbf{R}_{i} \leq 2000$ for all $1 \leq i \leq \mathbf{M}$

## Sample Test Cases

## Sample Input(s) (one in each test file)

```
20 2 3
10 11 11
20 2 5
9 8 7 4 0
20 2 2
10 0
1729 1000 10
0 0 0 1 1 1 2 2 3 4
```

Sample Output(s) (one in each test file)
1
0
748683265
520185694

## Sample Explanations

## Test File \#1:

In this case, the opposing Maushold hits twice, each with a $33.33 \%$ chance of dealing 10 HP and a $66.67 \%$ chance of dealing 11 HP. Since your Pokémon has only 20 HP, it will always faint, in other words, it will faint with probability 1.

## Test File \#2:

In this case, the opposing Maushold hits twice, each time with a $20 \%$ chance of dealing each number of damage between $9,8,7,4$, and 0 HP. Since your Pokémon has 20 HP , it will never faint (the maximum damage that Maushold can deal is 18 HP ), so the probability of fainting is just 0.

## Test File \#3:

In this case, the opposing Maushold hits twice, each time with a 50\% chance of dealing 10 HP and a $50 \%$ chance of dealing 0 HP. Since your Pokémon has 20 HP , the only case in which it will faint is if both hits deal 10 HP , so the probability of fainting is $1 / 4$. Since $4^{-1} \equiv 748683265$ $\bmod 998244353$, the answer is $1 \times 748683265=748683265$.

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