Problem 7: Is a configuration of blocks stable? 4+5 Points

Problem ID: `stableblocks`
Rank: `2+3`

Introduction

Peter Klotz lost his blocks. But rather than wishing for new ones, he asked Santa for a blocks simulator. Can you help Santa write one?

Problem Statement

Consider $\mathbf{N}$ rectangular blocks $B_0, B_1, \ldots, B_{\mathbf{N-1}}$ stacked upright in the 2D coordinate plane, with the floor represented by the line $y = 0$ (the x-axis). The blocks are axis-aligned and non-overlapping and you are given the lower left $(\mathbf{X0}_i, \mathbf{Y0}_i)$ and upper right $(\mathbf{X1}_i, \mathbf{Y1}_i)$ corner of each block. Furthermore, each block rests either on the floor ($\mathbf{Y0}_i = 0$) or on **exactly one other block**. Here, we say a block B_i is *resting* on another block B_j if the lower edge of B_i coincides with the upper edge of B_j.

Determine whether the overall structure of blocks is *stable*. That is, whether the positions of all blocks remain unchanged when left undisturbed under the force of gravity.

More precisely, we define the *substructure* associated with block B_i as a set containing B_i and all B_k for which there exists a sequence $B_i, B_j, \ldots, B_k$ where every block in the sequence rests on the block before it. A structure is stable if, for every block B_i, B_i is either resting on the floor or the center of gravity of its associated substructure lies between the lines extending the left and right edge of the block B_i rests on. See Figures 1, 2 and 3 for examples.

All blocks have constant density 1. Thus, the mass m_i of a block B_i is equal to its area:

$$m_i = (\mathbf{X1_i} - \mathbf{X0_i}) \cdot (\mathbf{Y1_i} - \mathbf{Y0_i})$$

The center of mass r_i of a block B_i is therefore obtained by averaging the coordinates of two opposite corners $(\mathbf{X0}_i, \mathbf{Y0}_i)$ and $(\mathbf{X1}_i, \mathbf{Y1}_i)$:

$$r_i = \left(\frac{\mathbf{X0}_i + \mathbf{X1}_i}{2}, \frac{\mathbf{Y0}_i + \mathbf{Y1}_i}{2} \right)$$

The center of mass r_{CM} of several blocks $B_i, B_j, \ldots, B_k$ is obtained by taking a average of their centers of mass $m_i, m_j, \ldots, m_k$, weighted by mass:

$$r_{\text{CM}} = \frac{m_i r_i + m_j r_j + \cdots + m_k r_k}{m_i + m_j \cdots + m_k}.$$

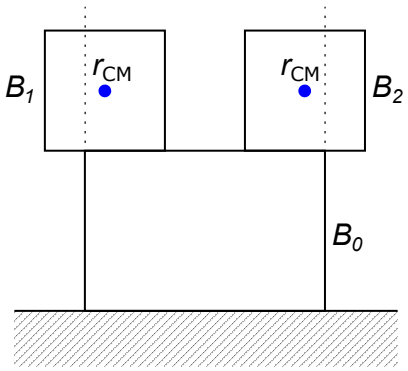
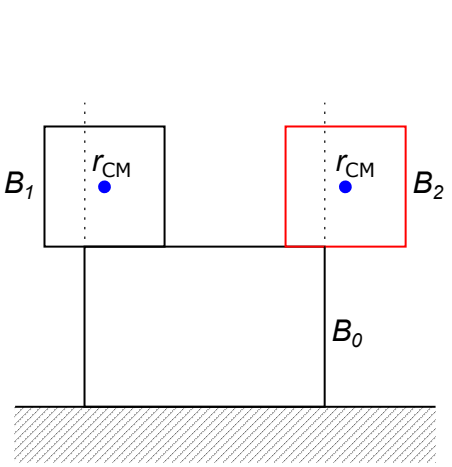
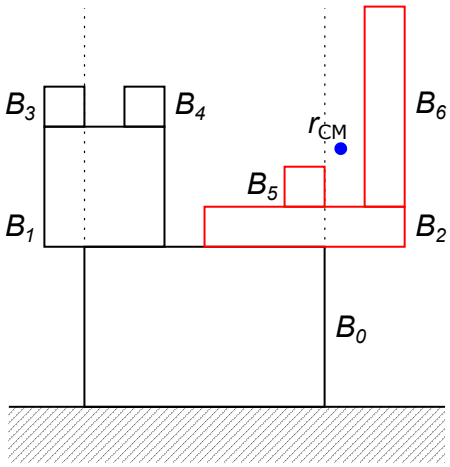


Figure 1: A stable configuration of blocks.



(a) Center of mass r_{cm} of B_2 not between left and right edge of B_0.



(b) Center of mass r_{cm} of blocks B_2, B_5 and B_6 not between left and right edge of B_0.

Figure 2: Unstable configurations of blocks.

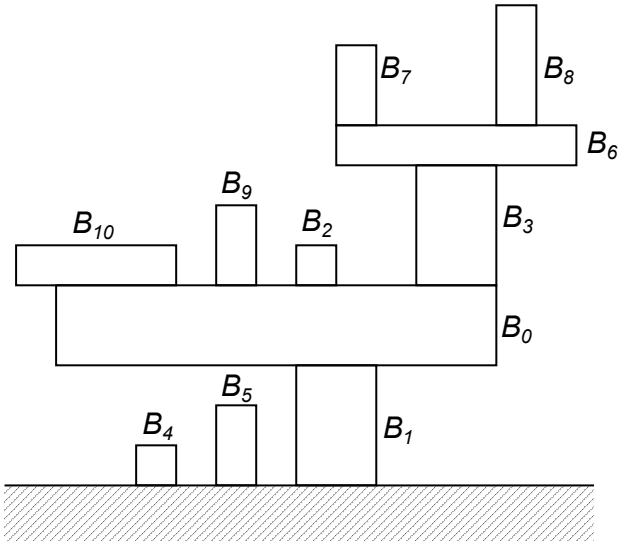


Figure 3: A stable configuration of blocks.

Input Format

The first line of the input contains a single integer $\mathbf{T}$ denoting the number of test cases that follow. For each test case:

- The first line contains a single integer $\mathbf{N}$ denoting the number of blocks.
- The next $\mathbf{N}$ lines each contain four space-separated integers $\mathbf{X0}_i$ $\mathbf{Y0}_i$ $\mathbf{X1}_i$ $\mathbf{Y1}_i$ with $\mathbf{X0}_i < \mathbf{X1}_i$ and $\mathbf{Y0}_i < \mathbf{Y1}_i$. These integers specify a rectangle with lower left corner $(\mathbf{X0}_i, \mathbf{Y0}_i)$ and upper right corner $(\mathbf{X1}_i, \mathbf{Y1}_i)$.

Output Format

For each test case, output a single string containing either the string `Stable` or `Unstable` based on whether the structure and substructures are stable or not.

Constraints

$1 \leq \mathbf{T} \leq 100$
$1 \leq \mathbf{N} \leq 10^5$
$-10^4 < \mathbf{X0}_i < \mathbf{X1}_i < 10^4$
$0 \leq \mathbf{Y0}_i < \mathbf{Y1}_i < 10^5$
The total area of all blocks in a test case does not exceed 10^5.
The total number of blocks across all test cases does not exceed 10^5.

Main Test Set

The blocks $B_0, \ldots, B_{\mathbf{N}-1}$ form a full binary tree of height h and are listed in level order (breadth first search order) from bottom to top. That is, $\mathbf{N} = 2^{h+1} - 1$ and blocks B_{2i+1} and B_{2i+2} are resting on block B_i for $i < 2^h - 1$.

Bonus Test Set

The blocks form any forest and are listed in any order.

Sample Test Cases

The main sample test cases correspond to Figure 1, 2a and 2b, respectively.

Main Sample Input Download

```
3
3
-3 0 3 4
-4 4 -1 7
1 4 4 7
3
-3 0 3 4
-4 4 -1 7
2 4 5 7
7
-3 0 3 4
-4 4 -1 7
0 4 5 5
-4 7 -3 8
-2 7 -1 8
2 5 3 6
4 5 5 10
```

Main Sample Output Download

```
Stable
Unstable
Unstable
```

The bonus sample test case corresponds to Figure 3

Bonus Sample Input Download

```
1
11
-6 3 5 5
0 0 2 3
0 5 1 6
3 5 5 8
-4 0 -3 1
-2 0 -1 2
1 8 7 9
1 9 2 11
5 9 6 12
-2 5 -1 7
-7 5 -3 6
```

Bonus Sample Output Download

```
Stable
```