Problem 4: Laser
3+6 Points

Problem ID: `laser`
Rank: `1+3`

Introduction

Self-proclaimed Planetary Defender-in-Training Big Ben is training up his planetary defense skills by shooting lasers at simulated asteroids on a plane. Sure, finding the asteroids is important, but what's the point of it all if you can't send them off with a great big bang?

Hopefully he doesn't miss and hit something else instead.

Problem Statement

Consider a section of the positive xy-plane, where $\mathbf{X_i}$ can take on discrete values between 0 and $\mathbf{N}-1$ (inclusive) and $\mathbf{Y_i}$ can take on discrete values between 0 and $\mathbf{M}-1$ (inclusive). You are given a list of $\mathbf{K}$ asteroids with position $(\mathbf{X_i}, \mathbf{Y_i})$ in the plane.

The position of the first asteroid denotes the starting position of the laser (although the laser will not hit the asteroid until it has wrapped around once). The laser moves in steps, traveling $\mathbf{P}$ units along the y-axis and $\mathbf{Q}$ units across the x-axis each step. The laser also wraps around the grid. If the laser moves such that it exits the top of the grid, it will appear at the bottom of the same column. Likewise, if the laser exits the right side of the grid, it will reappear on the left side of the same row.

Output the index of the first asteroid that the laser hits. Note that asteroids are points, and an asteroid is only hit if a laser intersects the point directly.

An example of how the lasers moves without asteroids is here. The visualization shows the case $\mathbf{N}=4$, $\mathbf{M}=3$, and $\mathbf{P}=\mathbf{Q}=1$.

*Note: Templates are available for this problem—and **all other problems in this contest**—in Python, Java, and C++! Find them in the contest.zip provided at the start of the contest. Templates handle input and output for you, so you can just fill out a single function!*

Input Format

The first line of the input contains a single integer $\mathbf{T}$ denoting the number of test cases that follow.

For each test case:

- The first line contains five space-separated integers $\mathbf{K}$ $\mathbf{N}$ $\mathbf{M}$ $\mathbf{P}$ $\mathbf{Q}$ denoting the number of asteroids, the dimensions of the grid, and the slope of the laser, respectively.
- The remaining $\mathbf{K}$ lines each contain a space-separated ordered pair $\mathbf{X_i}$ $\mathbf{Y_i}$ representing the coordinates of the asteroids. $\mathbf{X_i}$ denotes the position of the laser along the x-axis, and $\mathbf{Y_i}$ denotes the position of the laser along the y-axis.

Output Format

For each test case, output the index of the first asteroid which is hit (0 indexed).

Constraints

$1 \leq \mathbf{T} \leq 10$
$0 \leq \mathbf{X_i} \leq \mathbf{N} - 1$
$0 \leq \mathbf{Y_i} \leq \mathbf{M} - 1$
$\mathbf{P}$ and $\mathbf{Q}$ are guaranteed to be coprime.

Main Test Set

$1 \leq \mathbf{K} \leq 100$
$\mathbf{K} \leq \mathbf{N} \leq 10^3$
$\mathbf{K} \leq \mathbf{M} \leq 10^3$
$1 \leq \mathbf{P} \leq \mathbf{M}$
$1 \leq \mathbf{Q} \leq \mathbf{N}$

Bonus Test Set

$1 \leq \mathbf{K} \leq 10^5$
$\mathbf{K} \leq \mathbf{N} \leq 10^6$
$\mathbf{K} \leq \mathbf{M} \leq 10^6$
$1 \leq \mathbf{P} \leq \mathbf{M}$
$1 \leq \mathbf{Q} \leq \mathbf{N}$
It is guaranteed that the sum of $\mathbf{K}$ across all test cases is less than 10^5.

Sample Test Cases

Main Sample Input [Download]

```
4
3 10 10 1 1
0 0
2 2
9 9
3 6 8 2 1
1 1
1 5
5 1
3 6 8 2 1
1 1
2 3
4 4
3 5 5 2 3
2 2
0 0
1 4
```

Main Sample Output [Download]

```
1
2
1
0
```

Main Sample Explanations

Recall that the laser begins at the position of the first asteroid.

For test case #1, the laser starts at $(0, 0)$, and travels 1 unit along the y-axis and 1 units along the x-axis every step. After 2 steps, the laser will land on $(2, 2)$, meaning the asteroid at index 1 is the first to be hit by the laser.

For test case #2, the laser starts at $(1, 1)$, and moves to $(2, 3)$ after the first step. The fourth step would cause the laser to land on $(5, 9)$, which is outside the bounds of the grid; therefore the laser wraps to $(5, 1)$ instead. This is the position of the asteroid with index 2.

For test case #4, the laser starts at $(2, 2)$. It travels to the positions $(0, 4)$, $(3, 1)$,$(1, 3)$, $(4, 0)$, and finally $(2, 2)$, returning the laser to its starting position, which is the position of the asteroid with index 0. One can note in this case that it is not possible for the laser to hit any of the other two asteroids, no matter how many steps the laser is allowed to take.

Bonus Sample Input

Download

```
1
5 999999 1000000 2 1
0 0
500000 1
999997 0
499995 0
499995 1
```

Bonus Sample Output

Download

```
3
```

Bonus Sample Explanations

For test case #1, the laser will travel for $500,000$ steps before wrapping around to $(\texttt{500,000}, \texttt{0})$. After another $499,995,000,000$ steps, the laser will wrap around to $(\texttt{499,995}, \texttt{0})$, landing on the asteroid with index 3.