Problem 6: Shadow of the Clay
6 Points

Problem ID: `shadows`
Rank: `2`

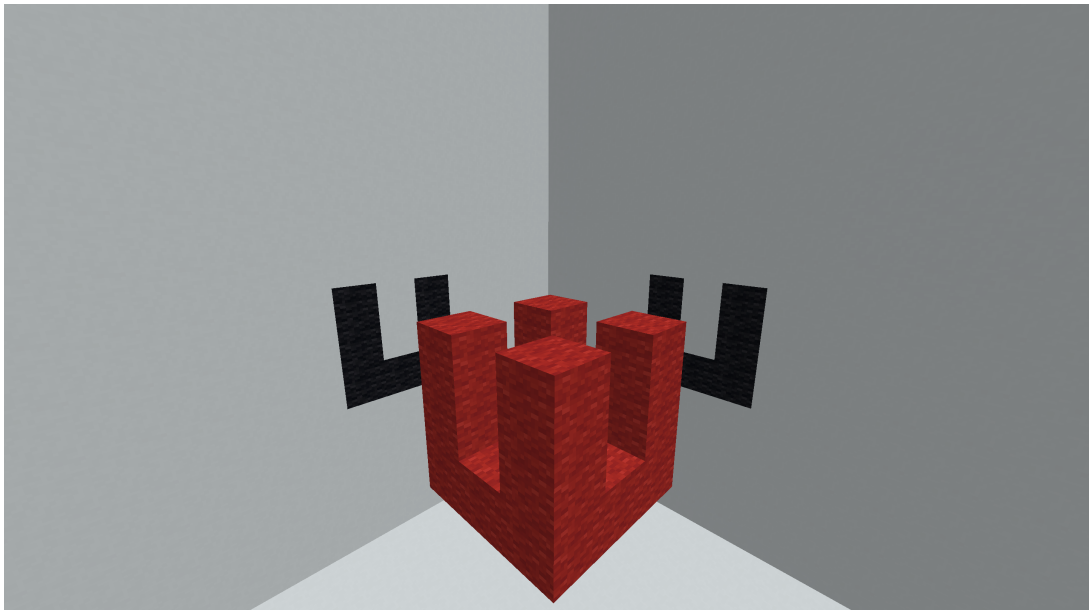
Introduction

Big Ben's friends have recently formed a garage rock band called LinkedIn Park, and Big Ben is eager to help. However, his friends have witnessed one too many of his karaoke sessions, and have instead asked him to make the cover art for their first album. Big Ben is on a tight budget, so he has decided to make some cool 3D figures out of clay, start a fire, and snap a few photos in his cave. After finishing the shoot, Big Ben realizes that due to camera mishaps, the shadows of the figure on the walls behind it are the only parts of the photo that are visible! Can you help him figure out what the original figure looked like?

Problem Statement

Consider a 3D shape contained inside a cube of dimensions $\mathbf{N} \times \mathbf{N} \times \mathbf{N}$. You are provided two 2D shadows (projections) of the shape represented by grids of dots `.` and hashtags `#`. The two shadows, denoted $\mathbf{S_1}$ and $\mathbf{S_2}$, are orthogonal to each other and parallel to the lateral faces of the cube.

In $\mathbf{S_1}$ and $\mathbf{S_2}$, the hashtags represent the portion of an $\mathbf{N} \times \mathbf{N}$ square that will be blocked by the 3D shape in the presence of a light source.



For the image above, $\mathbf{S_1}$ (seen on the left wall) is:

```
#.#
#.#
###
```

For the image above, $\mathbf{S_2}$ (seen on the right wall) is:

```
#.#
#.#
###
```

For some $\mathbf{S_1}$ and $\mathbf{S_2}$, it can be shown that there are multiple shapes which produce these shadows. Considering all such shapes, output the maximum and minimum volumes that are possible.

It is guaranteed that the shadows represent a valid shape and that each shape is made up of $1 \times 1 \times 1$ blocks. A shape does not need to be connected to be valid, i.e. it may be made up of multiple disjoint sets of blocks.

Input Format

The first line of the input contains a single integer $\mathbf{T}$ denoting the number of test cases that follow.

For each test case:

- The first line contains an integer $\mathbf{N}$ denoting the dimensions of the bounding cube.
- The next $\mathbf{N}$ lines each contain a string of length $\mathbf{N}$ representing a row of shadow $\mathbf{S_1}$.
- The next $\mathbf{N}$ lines each contain a string of length $\mathbf{N}$ representing a row of shadow $\mathbf{S_2}$.

Output Format

For each test case, output two space-separated integers representing the maximal and minimal volume of the shape, respectively.

Constraints

$1 \leq \mathbf{T} \leq 50$
$1 \leq \mathbf{N} \leq 10^3$
$\mathbf{S_1}$ and $\mathbf{S_2}$ will only contain the characters `.` and `#`.

Sample Test Cases

Sample Input Download

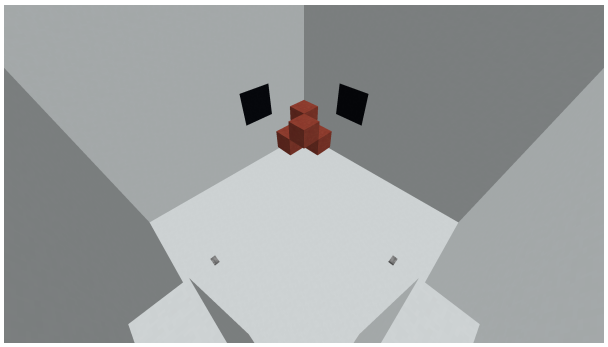
```
6
2
##
##
##
##
3
#.#
#.#
###
#.#
#.#
###
3
#.#
#.#
###
###
#.#
#.#
3
#.#
#.#
###
###
##.
##.
4
#...
..##
##.#
##..
..##
..##
##.#
#.##
2
..
..
..
..
```

Sample Output Download

```
8 4
17 7
16 8
16 8
21 10
0 0
```

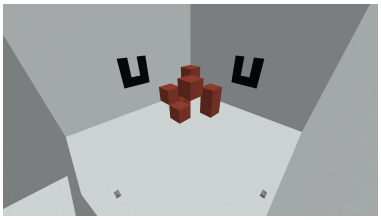
Main Sample Explanations

For test case #1, a $2 \times 2 \times 2$ cube would produce the provided shadows, for a volume of 8. A shape like the one shown below using only 4 blocks could also generate the shadows:

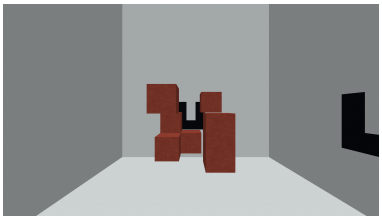


(a) Test Case #1 Min

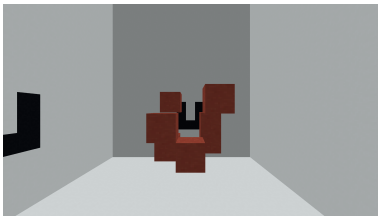
For test case #2, the shape with maximal volume is depicted in the problem statement. The shape with minimal volume is depicted below:



(a) Test Case #2 Min

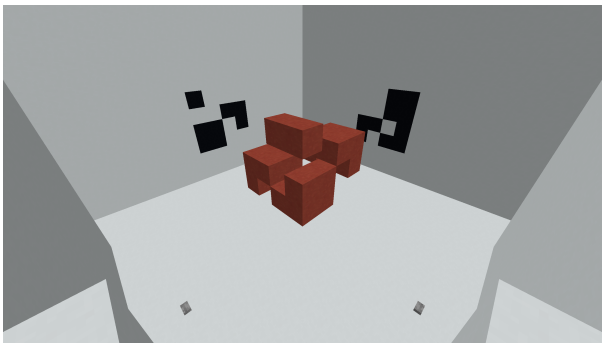


(b) Test Case #2 Min Left

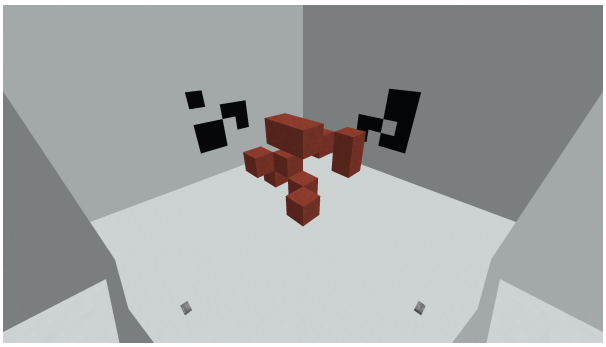


(c) Test Case #2 Min Right

For test case #5, a block configuration giving the maximum and minimum area is given below:



(a) Test Case #5 Max



(b) Test Case #5 Min